What is a feature: in vis/computational context

Structure computed from field, processed, visualized
Contrast with direct methods (glyphs, colormaps)

Colormaps, glyphs
volume rendering

(tensor) visualization

Features

Estimated Tensors

numercial methods,
tensor field analysis,
image processing,
non-local measurements,
etc...

visualization
What is a feature: in a biomedical context

Geometric model of anatomic structure important for study motivating the data acq.
Quantitative measurements of underlying biology
Pictures (visualization) may not be central

What is a feature: in a biomedical context

Feature definitions and considerations
Tractography and its clustering
Tractography methods
Clustering metrics and methods
Cluster representation and display
Segmentation from Tensor Distances
Differential Structure: Edges, Creases
Topological and Lagrangian Structure
Discussion
**Tractography (deterministic)**

Standard: Streamline integration of principle eigenvector

Stream tubes, Zhang *2003* TVCG 9:454-463


Basser ISMRM *1998*, MRM 44:625-63

**Tractography (Probabilistic)**

Explicitly represent uncertainty in path

Deterministic tractography \(\approx\) mode

Various uncertainties, relates to tensor model choice

Produces volume of connectivity values

From tensor fields:

Friman *2006* TMI 25:965-978

Sherbondy *2008* JoV 8:1-16
Tractography Clustering

Aims to create anatomically meaningful units
Starts with tractography pre-computation

Two ingredients: Distance, Clustering Algorithm

Inter-tract similarity → distance measures
Starts with tractography pre-computation

\[ F_i = \{p_k\} \]

\[ F_j = \{p_l\} \]

\[
\tilde{d}_\mu(F_i, F_j) = \text{mean}_{p_k \in F_i} \min_{p_l \in F_j} \| p_k - p_l \| \quad \text{(Euclidean distance)}
\]

\[
d_\mu(F_i, F_j) = \frac{1}{2} \left( \tilde{d}_\mu(F_i, F_j) + \tilde{d}_\mu(F_j, F_i) \right)
\]

\[
\tilde{d}_H(F_i, F_j) = \max_{p_k \in F_i} \min_{p_l \in F_j} \| p_k - p_l \| \quad \text{(Hausdorff distance)}
\]

\[
d_H(F_i, F_j) = \max \left( \tilde{d}_H(F_i, F_j), \tilde{d}_H(F_j, F_i) \right)
\]
Tractography Clustering, Algorithm

Range of possibilities

Nearest Neighbor (parameterized by distance D)
  Start w/ 1 tract/cluster
  Join clusters T, S if d(t_i,s_j) < D for t_i in T, s_j in S

Fuzzy Clustering (Maddah 2008 MIA 12:191-202)
Normalized Graph Cuts (Brun 2004 MICCAI-368–375)

Clustering parameters

  How many clusters?
  Interactive/Manual vs Automatic

Cluster Representation & Display

Representative or “Core” trajectories
  Reference curve for quantitative comparison

Shell or wrapper (Enders Vis 2005)

Rasterization to volumes-of-interest
  Easy integration with other segmentations
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Tensor Distance Measures

In support of segmentation

Creating geometric models of anatomy
Volumetric vs. surfaces
Euclidean: $t_{ij}$ or eigensystem

$$d_E(T^{(1)}, T^{(2)}) = \left( \sum_{i=1}^{3} \sum_{j=1}^{3} (t_{ij}^{(1)} - t_{ij}^{(2)})^2 \right)^{1/2}$$

$$\langle T^{(1)}, T^{(2)} \rangle = \sum_{i=1}^{3} \sum_{j=1}^{3} t_{ij}^{(1)} t_{ij}^{(2)} = \sum_{i=1}^{3} \sum_{j=1}^{3} \lambda_i^{(1)} \lambda_j^{(2)} (e_i^{(1)}, e_j^{(2)})^2$$

Non-Euclidian
Riemannian, Log-Euclidean
Geodesic-Loxodrome

allow shape or orientation-specific

How to evaluate distance measures?
Segmentation from Distances

Calculate volumetric regions (representing anatomy) based on distances between tensors at voxels

Challenge: low resolution

Wiegel NJ 2003 19:391-401

Level sets
Watershed
Region Models
Markov Random Fields
Graph-based
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Differential Structure: Edges
Gradient of tensor is 3rd order tensor
  Total magnitude: scalar
  Can be used for distinguishing regions
Pajevic 2002 JMR 154:85-100
  Decomposition of D into isotropic, deviatoric
Tensor derivative decomposition
Decomposition according to shape, orientation
Kindlmann 2007 TMI 26(11):1483-1499

(a) $|\nabla R_1| = |\nabla D|$
(b) $|\nabla R_2| = |\nabla FA|$
(c) $|\nabla R_3| = |\nabla \text{mode}|$

Tensor derivative decomposition
Decomposition according to shape, orientation
Kindlmann 2007 TMI 26(11):1483-1499

(a) $\text{RGB}(e_1)$
(b) $|\nabla \bar{R}_3|$

(e) $|\nabla \bar{R}_3|$
(d) $AO = \sqrt{|\nabla R_1|^2 + |\nabla \phi_2|^2}$
Tensor derivative decomposition

Decomposition according to shape, orientation
Kindlmann 2007 TMI 26(11):1483-1499

Differential Structure: Creases

Ridges & Valleys: “Creases”
For DTI: creases of tensor invariants, like FA

“Ridges in Image and Data Analysis” Eberly ’96
Constrained extremum
Gradient $\mathbf{g}$
Hessian eigensystem $\mathbf{e}_i, \lambda_i$

**Crease**: $\mathbf{g}$ orthogonal to one or more $\mathbf{e}_i$

Eigenvalue gives **strength**

Ridge surface: $\mathbf{g} \cdot \mathbf{e}_3 = 0$; $\lambda_3 < \text{thresh}$
Ridge line: $\mathbf{g} \cdot \mathbf{e}_3 = \mathbf{g} \cdot \mathbf{e}_2 = 0$; $\lambda_3, \lambda_2 < \text{thresh}$
Valley surface: $\mathbf{g} \cdot \mathbf{e}_1 = 0$; $\lambda_1 > \text{thresh}$
**FA ridges surfaces**

Studied in both Vis and biomedical areas

Kindlmann 2007 *MIA* 11:492-502

Kindlmann 2009 *15:1415-1424*

Smith 2006 *NI* 31:1487-1505

Why not connectivity?

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**Stream surfaces**

Analogy to streamlines, propagate surface along medium and minor eigenvectors

Schultz 2010 *TVCG* 16:109-119:
Surface depends on visit order
→ ridge surfaces of planar anisotropy

Surfaces for areas of planarity

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Tensor field topology

Based on definitions from vector field topology
Loci of points of tensor eigenvalue equality
Genericity considerations $\rightarrow$ lines (co-dim 2)
Expressible as crease lines of tensor mode
Poor anatomic relevance
Lagrangian Coherent Structure

Quantifies stability of tractography WRT seedpoint
A non-local gradient measure

Hlawitschka 2010 JCARS 5(2):125–131

Discussion

Dynamic mix of DTI analysis methods
  From Visualization: Tractography
  From Machine Learning: Clustering
  From Vision: Edges and Creases
  From Dynamical Systems: LCS

Interplay between theory and biomedicine
  Math structure may or may not be anatomical
  Standards for evaluation are complex

Visualization can have scientific impact
Thank you!
Questions: glk@uchicago.edu