Proximity Graphs for Defining Surfaces over Point Clouds

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Surface Definition based on MLS

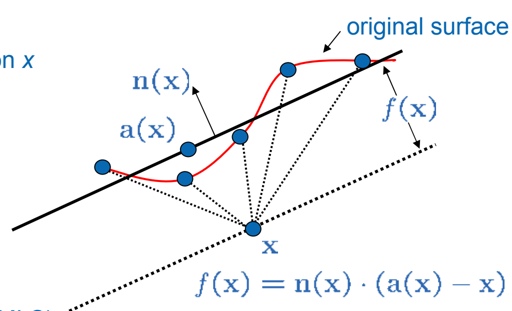
- Surface can be approximated by implicit function, i.e.,

$$S = \{x : f(x) = 0, x \in \mathbb{R}^3\}$$

$\mathrm{a}(\mathrm{x})$: weighted average of points at location *x*

$$\mathrm{a}(\mathrm{x}) = \frac{\sum_{i=1}^{N} \theta(\|\mathrm{x} - \mathrm{p}_i\|)\mathrm{p}_i}{\sum_{i=1}^{N} \theta(\|\mathrm{x} - \mathrm{p}_i\|)}$$

$$\theta(d) = e^{-d^2/h^2}$$ ← kernel bandwidth



n is determined by moving least squares (MLS):

Minimize $\sum_{i=1}^{N} (\mathrm{n}(\mathrm{x}) \cdot (\mathrm{a}(\mathrm{x}) - \mathrm{p}_i))^2 \theta(\|\mathrm{x} - \mathrm{p}_i\|)$ for fixed x and ||n(x)||=1.

Artifacts

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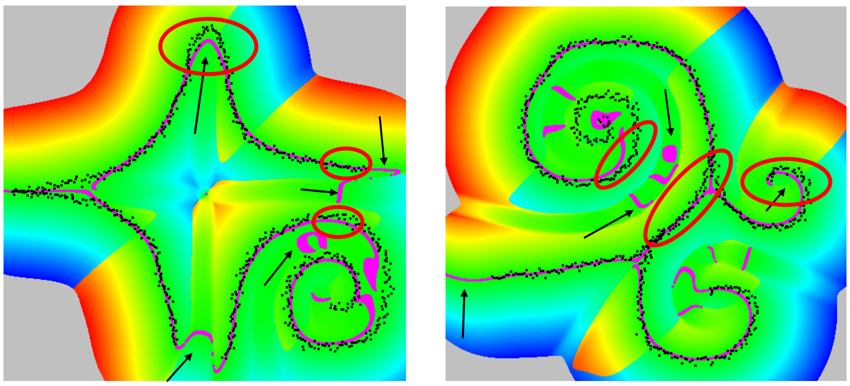
Surface reconstruction based on conventional MLS can suffer from artifacts:

1. Unwanted zero sets
2. Bias

(Artifacts at borders have already been dealt with [Adamson & Alexa, 2004]).



Related Work

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- Approximating and Intersecting Surfaces from Points [Adamson & Alexa, 2003]
- Curve Reconstruction from Unorganized Points [Lee, 2000]
- Multi-level Partition of Unity Implicits [Ohtake et al., 2003]
- Smooth Surface Reconstruction via Natural Neighbour Interpolation of Distance Functions [Boissonnat & Cazals, 2000]

Our Contribution

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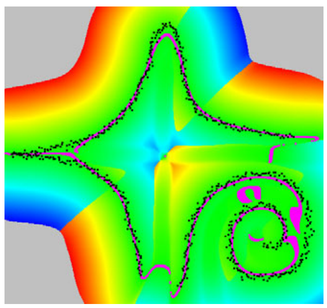
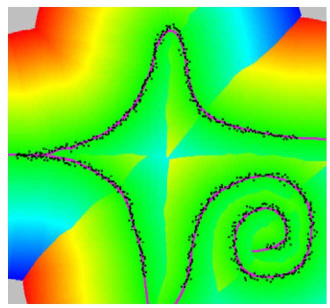
- New definition of an implicit surface over a (noisy) point cloud.
- New kernel that approximates geodesic distances using a proximity graph.
- The performance is of the same order as that of the Euclidean kernel.
- Artifacts and root mean square error are significantly reduced.



Euclidean kernel



our new geodesic kernel

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New Point Cloud Surface Definition

Geodesic Distance Approximation

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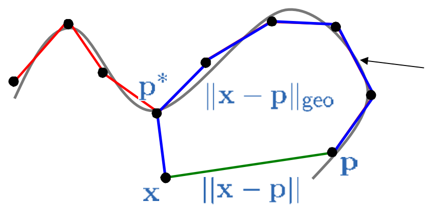
- Take topology of *S* into account → approximate <u>geodesic</u> distances on *S*.
- Use a proximity graph where nodes are points in *P*:
  - compute nearest neighbor p* of x.
  - *d*(p*,p) shortest path in graph from p* to p.

$$\|\mathbf{x}-\mathbf{p}\|_{\text{geo}} = \|\mathbf{x}-\mathbf{p}^*\| + d(\mathbf{p}^*,\mathbf{p})$$



Close-Pairs Shortest-Paths (CPSP)

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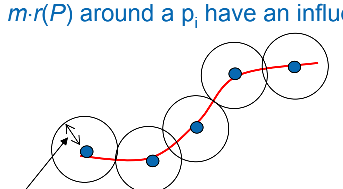
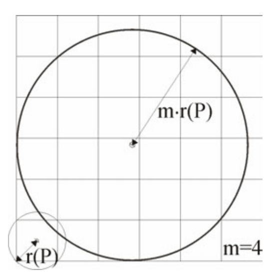
- CPSP, which is a subset of APSP, can be precomputed in O(N).
- $\theta(d)$ decays quickly → compute only paths up to some length *r*.
- For each p_i: SSSP for all points in sphere S_i around p_i with radius *r*.
- If points are uniformly distributed and *r* =m·r(P) → O(1) points in S_i.

h should be chosen so that points up to a distance *m*·*r*(*P*) around a p_i have an influence [KZ04].



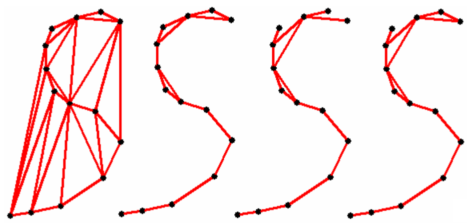
sampling radius r(P)



sphere with radius m·r(P) can be covered by O(m³) spheres with radius r(P) [Rog63]

Proximity by Delaunay Graph

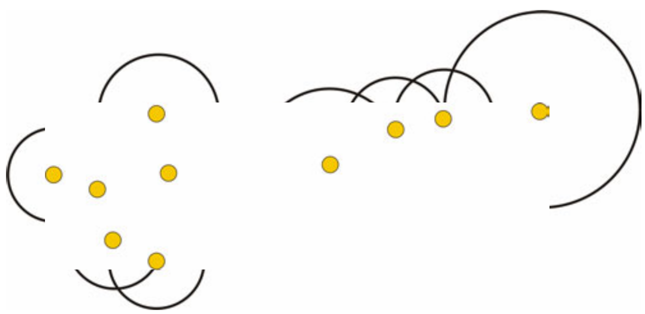
- Geodesic distances can be approximated by shortest paths on edges of DG(*P*).
- Some edges can "tunnel" through space.



- Prune edges longer than global sampling density or use an outlier detection algorithm.
- Best results are achieved by pruning edges with length > Q_2 (median).

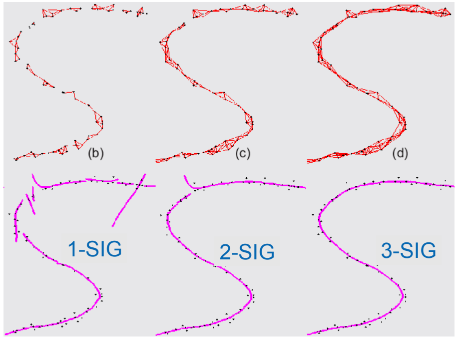
Proximity by Sphere-of-Influence Graph

- SIG is sparse: < 18·N edges (N = # nodes).
- SIG is not a subgraph of DG.
- Radius of influence: $r_i = ||p_i - \text{nearest}(p_i)||$.
- p_i and p_j are connected, if $||p_i – p_j|| \leq r_i + r_j$ $\rightarrow$ connect points that are locally close.



r-SIG

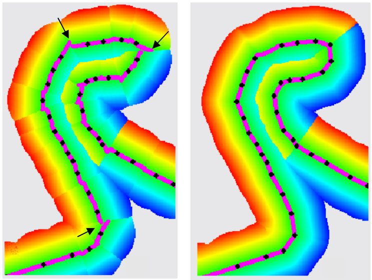
- Problems with simple SIG in noisy or irregularly sampled point clouds:
  - "mini-clusters"
  - "detours"
- Consequence:
  approximate geodesic distances are imprecise, because
  - close points can only indirectly be accessed
  - some points are not considered due to unconnected components.



- Our solution:
  r-th order SIG: use sphere determined by r-nearest neighbor.

Reducing Discontinuities

- Discontinuities in *f* and *S* can occur at borders of Voronoi regions.
- $||x_1 - x_2|| \approx 0$, but $||p^*(x_1) - p^*(x_2)||$ is "large".



$$\|\mathrm{x} - \mathrm{p}\|_{\mathrm{geo}(k)} = \min_{p^* \in \mathcal{P}_k(\mathrm{x})} \{\|\mathrm{x} - \mathrm{p}^*\| + d(\mathrm{p}^*, \mathrm{p})\}$$

$\mathcal{P}_k(\mathrm{x})$ = k-nearest neighbors of x that are reachable from nearest neighbor.

Complexity for the 3D Case

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- Pre-computations (under reasonable assumptions):
  - DG(P) can be determined in O(*N*) [Attali & Boissonnat, 2002]
  - r-SIG(*P*) can be determined in O(*N*) on average [Dwyer, 1995]
  - CPSP can be determined in O(*N*).
- Run-time:
  - Nearest neighbor in O(log *N*) by a Delaunay hierarchy [Devillers, 2002].
  - BFS allows to determine all points in a sphere around p* of radius m·r(P) in O(1).

  → f(x) can be evaluated in O(log *N*)

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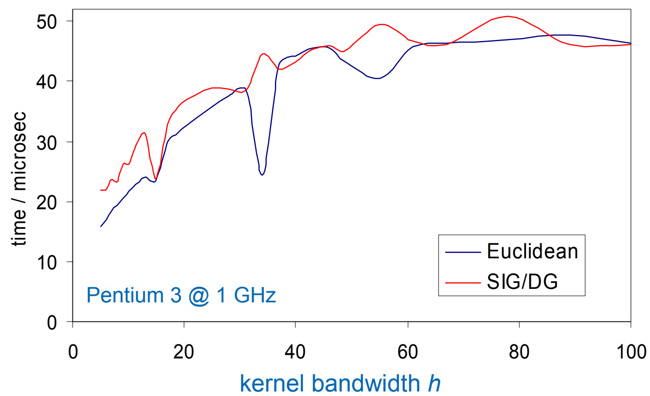
Results

Fast Evaluation

- The performance for both kernels is of the same order.



Quality

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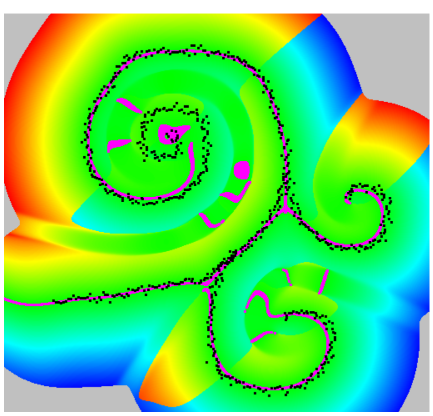
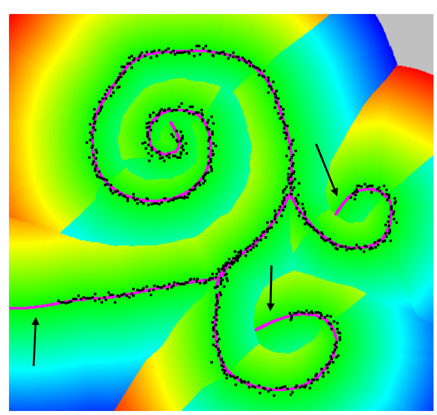
- Artifacts can be reduced *significantly*.



Euclidean kernel



geodesic kernel

RMSE depending on Bandwidth

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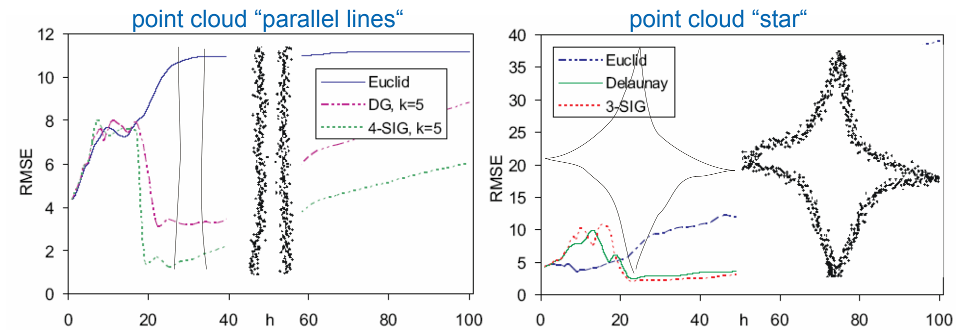
- RMSE: $\sqrt{\frac{1}{|\hat{S}|}\sum_{x\in\hat{S}} f(x)^2}$ $\hat{S}$ is sampling of the original surface.
- For nearly all h, our new kernel yields a smaller RMSE.
- New kernel is less sensitive towards *h*.
- SIG seems to achieve better results in several examples.



RMSE depending on k and r

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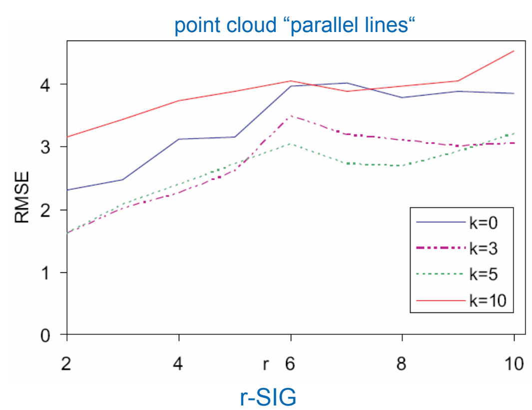
- Two parameters k and r ($\|\ldots\|_{geo(k)}$ and r-SIG) are very robust.
- If k and r are $\in$ [3…6], very similar results are achieved.



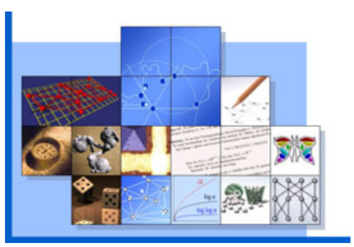
Conclusions and Future Work

- Our new definition of implicit functions over noisy point clouds yields surfaces that are much closer to the original surface.
- Less artifacts
- Auxiliary data structures can be constructed efficiently and consume only O(*N*) space.
- f(x) can be evaluated fast

Future Work

- Models with boundaries.
- Deformable point clouds.
- Automatic detection of kernel bandwidth by proximity graph.

Thank you!



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