Accurate Interactive Visualization of Large Deformations and Variability in Biomedical Image Ensembles

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Navigate shape space

- Manual (e.g. via linked scatter plot)
- Regression (parametrize labeled attribute)
- Classification (group difference)
- Direct manipulation

Blanz & Vetter
/Siggraph 1998

Hermann et al.
/PacificVis 2014

Busking et al./EuroVis 2010

Klemm et al./VAST 2014
Shape analysis and deformations

- Linear combinations of principal deformation modes
Synthesis comparison

template

linear

SVF

baseline

RMSE: $10.4 \times 10^{-3}$
FPS: 22

4.6$ \times 10^{-3}$
4

0 (baseline)
N/A (offline)
Overview

- **Methods:**
  - Log-domain 3D image warping
- **Applications:**
  - Browsing group mean shapes
  - Volumetric reformation
  - Projected streamlines
  - Likelihood volumes
- **Case study w/ morphometric expert**
Method:
Log-domain 3D image warping
Image warping

• Forward deformation $\varphi(x) = x + u(x)$ with vector field $u$

\[\varphi\]

• Discrete image domain $\Rightarrow$ scattered data interpolation

• Standard approach: Use **backwards deformation** $\varphi^{-1}$

\[I(x) = \bar{I} \circ \varphi^{-1}(x)\]

[Heckbert1989]
Direct space warping

- Apply inverse deformation along viewing ray

- Approximation* \( \tilde{\phi}^{-1}(x) = x - u(x) \)
- \( \tilde{\phi}^{-1}(\varphi(x)) = x + \sigma(\tau^2) \) for maximum deformation magnitude \( \tau \)
Diffeomorphisms

- Deformations in computational anatomy are **diffeomorphic**
- Transport equation
  \[
  \frac{d}{dt} \phi(x, t) = w(\phi(x, t), t)
  \]
  with initial condition \( \phi(x, 0) = x \) and \( \varphi(x) = \phi(x, 1) \)
- Large deformation diffeomorphic metric mappings (LDDMM)
- Time-varying flow \( w \) computationally expensive!

[Beg et al. 2005]
[Sotiras et al./MI2013]
Stationary velocity fields

- Deformations in computational anatomy are **diffeomorphic**
- Stationary parametrization

\[
\frac{d}{dt} \phi(x, t) = v(\phi(x, t))
\]

with initial condition \( \phi(x, 0) = x \) and \( \varphi(x) = \phi(x, 1) \)

- **Stationary velocity field (SVF) \( v \)**
- Generator of one-parameter subgroup

[ Arsingy 2006 ]
[ Ashburner 2007 ]
[ Vercauteren et al. 2008 ]
Log-domain

• Exponential map

\[ \varphi(x) = \exp(v)(x) := x + \int_{0}^{1} v(\varphi(x,t)) dt \]

• Logarithm

\[ v = \log(\varphi) \iff \exp(v) = \varphi \]

• Log-Euclidean calculus

\[ \varphi_a \circ \varphi_b = \exp(v_a + v_b) \]

\[ \varphi^{-1} = \exp(-v) \]

\[ \varphi \text{ close enough to } \text{Id} \]

\[ v \text{ in a neighbourhood of } 0 \]

[Arsigny et al. 2006]
Shader implementation

• Solve by numerical integration

\[ \varphi(x) = \exp(v)(x) := x + \int_{0}^{1} v(\phi(x, t))dt \]

Texture lookups

13

Position along casted ray forward/backward

Texture lookups

```cpp
vec3 integrate(vec3 x0, int steps, float sign) {
    vec3 x = x0;
    float h = sign / (float)steps; // Step size
    for (int k=0; k < steps; k++) {
        // Runge-Kutta (RK4) steps
        vec3 k1 = h * get_velocity(x);
        vec3 k2 = h * get_velocity(x + 0.5*k1);
        vec3 k3 = h * get_velocity(x + 0.5*k2);
        vec3 k4 = h * get_velocity(x + k3);
        x = x + k1/6.0 + k2/3.0 + k3/3.0 + k4/6.0;
    }
    return (x-x0); // Return displacement
}
```
Quality

• Compare rendering GPU vs. offline

• Synthesis/reconstruction of 16 individuals
Application:
Browse group mean shapes
Summary

• Log-domain 3D image warping
• Improved interactive shape analysis tools

Future work

• Scale to large ensembles (n>100)
• Extend to multiscale shape model
• Interactive image registration
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Performance

• Framerate on a Nvidia GeForce GTX 780

*) integration with two steps
Statistical deformation model

- Linear model based on $\varphi_i(x) = x + u_i(x)$ with $u_i \in \mathbb{R}^{3 \times \text{#voxels}}$
- Mean $\bar{u} = \frac{1}{n} \sum_{i=1}^{n} u_i = 0$ by choice of template
- Covariance $\Sigma = \frac{1}{n-1} u_i u_i^T$, $\text{dim}(\Sigma) = n' \leq n$
- Basis $B$ for **generative model** via PCA
  $$u = Bc, \quad p(c) = (2\pi)^{\frac{n'}{2}} e^{-\frac{1}{2} \|c\|}$$

[Rueckert et al. 2001]
Non-linear deformation model

- Our model is based on $\mathbf{v}_i = \log(\varphi_i)$ with $\mathbf{v}_i \in \mathbb{R}^{3 \times \text{#voxels}}$
- Mean $\bar{\mathbf{v}} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{v}_i = 0$ by choice of template
- Covariance $\Sigma = \frac{1}{n-1} \mathbf{v}_i \mathbf{v}_i^T$ dim$(\Sigma) = n' \leq n$
- Basis $B$ for generative model via PCA

$$\mathbf{v} = Bc, \quad p(c) = (2\pi)^{\frac{n'}{2}} e^{-\frac{1}{2}||c||}$$
Log-Euclidean calculus (finite dimensional Lie groups)

- Matrix groups, e.g. SO(3)
  - Interpolation of rotations, inverse kinematics
  - Averaging rotations for camera calibration [Manton et al. 2004]
  - SPD metrics
- Shape analysis (for m-Rep‘s)
  - Principal geodesic analysis [Fletcher et al. 2004]